B.Sc. Physics (Hons) Part I

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Q Obtain Poissenuill's equation for the flow of liquid through a narrow tube with the necessary length correction.?

Ans Let us consider a narrow tube of radius 'r' through which liquid flows in a stream line motion. Let us take a cylindrical layer at a distance 'x' from the axis. Let the velocity of flow of liquid at this level be V over this layer of liquid due to outer layer of liquid. There will be a backward dragging force. The magnitude will be $\eta\, 2\pi x l \frac{dv}{dx}$, where $2\pi x l$ will be the area of the layer 'l' being the length of the tube and $\frac{dv}{dx}$ is velocity gradient. Let the difference of pressure between the two ends of the tube be P. Thus the forward force acting on the cylindrical layer will be $P \cdot \pi x^2$. For steady flow those two forces should be equal and opposite.

$$P\pi x^2 = -\eta\, 2\pi x l \frac{dv}{dx} \qquad \text{------ (I)}$$

$$dv = -\frac{\pi x^2 P \cdot dx}{2\pi x \eta l} = -\frac{P x\, dx}{2\eta l}$$

on integrating, $V = \frac{-P}{2\eta l}\frac{x^2}{2} + c$

At $x = r$, $V = 0$

$$\therefore 0 = \frac{-Pr^2}{4\eta l} + c \qquad \therefore c = \frac{Pr^2}{4\eta l} \qquad \text{------ (II)}$$

$$\therefore V = \frac{Pr^2}{4\eta l} - \frac{Px^2}{4\eta l} = \frac{P}{4\eta l}(r^2 - x^2) \qquad \text{------ (III)}$$

So this will be the, velocity of the liquid flowing a distance x from the axis.

Now if we imagine another co-axial layer at a distance dx, then the cross sectional area between the two layer will be $2\pi x dx$. Therefore flow of liquid per second through this area will be $2\pi x dx V = dv$

$$\therefore dv = 2\pi x dx \cdot \frac{P}{4\eta l}(r^2 - x^2) \qquad \text{------ (IV)}$$

$$= \frac{\pi P(r^2 - x^2)}{2\eta l}\, x \cdot dx$$

Total volume flowing through the tube per second

$$V = \int_0^r \frac{\pi P(r^2 - x^2)}{2\eta l}\, x \cdot dx$$

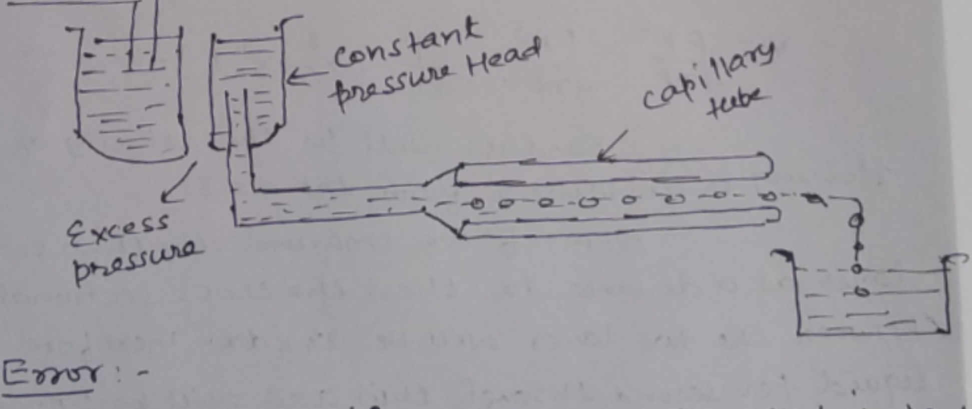
$$= \frac{\pi P}{2\eta l}\int_0^r (r^2 x - x^3)\, dx \Rightarrow \frac{\pi P}{2\eta l}\left[\frac{r^2 x^2}{2} - \frac{x^4}{4}\right]_0^r$$

$$V = \frac{\pi P}{2\eta l}\left[\frac{r^4}{2} - \frac{r^4}{4}\right] = \frac{\pi P r^4}{8\eta l} \quad \text{------} \quad (V)$$

$$\therefore \eta = \frac{\pi P r^4}{8\eta l} \quad \text{---}$$

By knowing P, r, V and l, 'η' can be obtained.

A ~~Ho~~ horizontal capillary tube is connected to a constant pressure head. In a pressure head liquid comes in it by the external or main tap and with other tube liquid goes to capillary tube, the height of the pressure head can be adjusted to given a particular difference between the two ends of the capillary tube. The liquid flowing steadily in the tube is collected in weighted beaker for a known time. So mass flowing of the liquid per unit time, hence volume flowing of the liquid per unit time can be determined length of the capillary tube measured carefully. Either directly by microscope the radius of tube is obtained by travelling microscope.



Error :-

The pressure difference between two ends is partly utilised in supplying K.E to following liquid